EXERCISE – IV

HINTS & SOLUTIONS

Sol.1 $E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$F = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$EF = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$FE = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E^2F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$FE^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$E^2F + FE^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = E$

Sol.2 (i) a_{ij} is 1 or –1
(ii) $a_{11}{}^2 + a_{12}{}^2 = a_{21}{}^2 + a_{22}{}^2 = 2$
(iii) $a_1, a_2, + a_{12} + a_{22} = 0$
Total no of matrix with condition (i) = 16
Total no of matrix with condition (ii) = 16
Total no of matrix with condition (iii) = 8
Total no of matrix satisfying all three condition=8

Sol.3 $\begin{bmatrix} 3 & -2 \\ 3 & 0 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} y & y \\ x & x \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3y & 3y \\ 10 & 10 \end{bmatrix}$

$\begin{bmatrix} 3y-2x & 3y-2x \\ 3y & 3y \\ 2y+4x & 2y+4x \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3y & 3y \\ 10 & 10 \end{bmatrix}$

$3y – 2x = 3$
$2y + 4x = 10$
$x = 3/2$
$y = 2$

Sol.4 $\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\varphi & \sin\varphi\cos\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta\cos^2\varphi + \sin\theta\cos\theta\cos\varphi\sin\varphi & \cos^2\theta\sin\varphi\cos\varphi + \sin\theta\cos\theta\sin^2\varphi \\ \cos\theta\sin\theta\cos^2\varphi + \sin^2\theta\cos\varphi\sin\varphi & \cos\theta\sin\theta\sin\varphi\cos\varphi + \sin^2\varphi\sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a nullmatrix

so, $\cos^2\theta\cos^2\phi + \sin\theta\cos\theta\cos\phi\sin\phi = 0$
$\cos\theta\cos\phi(\cos\theta\cos\phi + \sin\theta\sin\phi) = 0$
$\cos(\theta - \phi) = 0$

$\cos(\theta - \phi) = \cos(2n+1)\frac{\pi}{2}$

$\theta - \phi = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{2} + \phi$

Sol.5 $A^2 = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$A^4 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ $\quad (A^8 + A^6 + A^4 + A^2 + I)\, V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$

$A^6 = \begin{bmatrix} 27 & 0 \\ 0 & 27 \end{bmatrix}$

$A^8 = \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\left[\begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix} + \begin{bmatrix} 27 & 0 \\ 0 & 27 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} 121 & 0 \\ 0 & 121 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$

$\begin{bmatrix} 121x \\ 121y \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$

$x = 0$
$y = 1/11$

$v = \begin{bmatrix} 0 \\ 1/11 \end{bmatrix}$

Sol.6 $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$f(x) = x^3 - 6x^2 + 7x + 2$
$f(A) = A^3 - 6A^2 + 7A + 2$

$$A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 49 \end{bmatrix}$$

$$F(A) = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 49 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$+7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Sol.7 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

AB = BA

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

$a + 2c = a + 3b$ ......(1)
$2c = 3b$ ......(2)
$d = a + c$ ......(3)
From (iii)

$$\frac{a-b}{a+c-b} = \frac{a+c-b}{a+c-b} = 1$$

Sol.8 $\begin{bmatrix} a & b \\ c & 1-b \end{bmatrix}$

$A^2 = A$

$$= \begin{bmatrix} a & b \\ c & 1-b \end{bmatrix}\begin{bmatrix} a & b \\ c & 1-b \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc & ab+cb(1-b) \\ ca+(1-b)c & bc+(1-b)^2 \end{bmatrix}$$

$a = a^2 + bc$ .....(1)
$ab + b - b^2 = b$
$b^2 = ab$ .....(2)
$ca + c - bc = c$
$ca = cb$
$a = b$
$1 - b = bc + (1 - b^2)$
$a = a^2 + 1/4$ $f(x) = x - x^2$
$a = a^2 + 1/4$ $f(a) = 1/2 - (1/2)^2$
$(a - 1/2)^2 = 0$ $= 1/4$
$(a = 1/2)$

Sol.9 $A^2 = I$ Involutary matrix

$= 1/2\,(I + A)\ \frac{1}{2}\ (I + A)$

$= \frac{1}{4}\ [I^2 + IA + AI + A^2]$

$= \frac{1}{2}\ [I + A]$ Idempotent matrix

$\frac{1}{2}(I - A)\ \frac{1}{2}(I - A) = 0$

$\frac{1}{4}(I - A)\ (I - A) = \frac{1}{4}[I^2 - A^2 + AI - A^2]$ $[A^2 = I]$

$= 0$

Sol.10 $A A^T = I$

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}\begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix}$$

$$= \begin{bmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$4\beta^2 + \gamma^2 = 1$ .....(i)
$\alpha^2 + \beta^2 + \gamma^2 = 1$ .....(ii)
$a^2 - b^2 - g^2 = 0$ ....(iii)
From (i), (ii), (iii), (iv)

$\alpha = \pm 1/\sqrt{2}$

$b = \pm 1/\sqrt{6}$

$g = \pm 1/\sqrt{3}$

Sol.11 $A = \begin{bmatrix} 1 & x & 1 \\ x & 2 & y \\ 1 & y & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -3 & z \\ -3 & 2 & -3 \\ 2 & -3 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & x & 1 \\ x & 2 & y \\ 1 & y & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 & 2 \\ -3 & 2 & -3 \\ 2 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3x+z & -3+2x+3 & z+3x+1 \\ 3x-6+yz & -3x+4-3y & xz+6+y \\ 3-3y+3z & -3+2y-9 & 2+3y+3 \end{bmatrix}$$

AB is symmetric matrix so

$-6 + 2x = 3x - 6 + yz$ .....(1)

$x + yz = 0$ .....(2)

$3x + 3y - 2z = 2$

$xz - y = -6$ .....(3)

$$\left(-\frac{4\sqrt{2}}{3}, \frac{2}{3}, 2\sqrt{2}\right), \left(\frac{4\sqrt{2}}{3}, \frac{2}{3}, -2\sqrt{2}\right), (3,\ 3, -1)$$

Sol.12 $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & 3 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2 = I$

$A^2 = I$

A^6 ....

S $\cos x\ \theta + \sin x\ \theta$

$\cos^2\theta + \cos^4\theta + \cos^6\theta + ...+ \infty + \sin^2\theta \tan^4\theta$

$+ \sin^6\theta + ...\infty$

$$= \frac{\cos^2\theta}{1-\cos^2\theta} + \frac{\sin^2\theta}{1-\sin^2\theta}$$

$= \cot^2\theta + \tan^2\theta$

Minimum value = 2

Sol.13 $(AB)^T = B^T A^T$

$A \begin{bmatrix} 1 & 2 \\ 2 & -3 \\ -1 & 2 \end{bmatrix}$ and $B^T = \begin{bmatrix} 2 & -3 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & -3 \\ -1 & 2 \end{bmatrix}_{2\times 3} \begin{bmatrix} 2 & -3 & 5 \\ 1 & 2 & 3 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 2+2 & -3+4 & 5+6 \\ 4-3 & -6-6 & 10-9 \\ -2+2 & -3+4 & -5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & 11 \\ 1 & -12 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 4 & 1 & 11 \\ 1 & -12 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 2 & 1 \\ -3 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & 0 \\ 1 & -12 & 1 \\ 11 & 1 & 1 \end{bmatrix}$$

Sol.14 $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & -6 \\ -1 & 0 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ -1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 5 \\ 0 & 0 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

lower triangular matrix — upper triangular matrix in leading diagonal

symmetric matrix

$A = \left[\frac{A+A}{2}\right] + \left[\frac{A+A}{2}\right]$ skew symmetric matrix

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -3 \\ 2 & -3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & -3 \\ -3 & 3 & 0 \end{bmatrix}$$

Sol.15 $\lambda = \frac{n(n+1)}{2} + 1$

m = n + 1

$p = \frac{n^2 - n}{2}$

$\lambda + 5 = P + 2m$

$\frac{n(n+1)}{2} + 1 + 5 = \frac{n^2 - n}{2} + 2n + 2$

n = 4

Sol.16 $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

If C = (AB) (BTA)
D = (BTA) (AB)

n(A) denotes the number of elements in a such that (xy) non confimatmation = 0

order of c = 2 × 2
so number of elements 14 C = 4
h(c) = 4
D = (BTA) (AB)
Quadratic of D = | x |
D(1)t = 1
D = [18]
| D | = 18

$= \frac{n(c)\,|D|^2 + h(D)}{h(A) - h(B)}$

$= \frac{4((18)^2 - 1)}{4 - 2} = 650$

Sol.17 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^2 + FA + gI = 0$

$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ca + dc & cd + d^2 \end{bmatrix}$

f = (A)
= – (a + d)
g = (ad – bc)

$\begin{bmatrix} a^2 + bc & ab + bd \\ ca + dc & cd + d^2 \end{bmatrix} - (a + d) \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$+ ad - bc \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol.18 |A| = a
B = adj (A)
$|B| = |A|^{n-1}$
$= |A|^{3-1}$
$= |A|^2$
$= a^2 = b^-$

$\frac{1}{2} S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} \ldots\ldots \infty$

$\frac{1}{2} S = \frac{a}{b^2} + \frac{a^2}{b^6} + \frac{a^3}{b^{10}} \ldots\ldots \infty$

$= 1/a + 1/a^4 + 1/4^7 + \ldots\ldots \infty$
$= 1/3 + 1/81 + 1/(3)^7 + \ldots\ldots \infty$
common ratio = 1/27

$S_\infty = \frac{1/3}{1 - 1/27} = \frac{9}{26}$

$1/2\ S = S\infty$

$S = \frac{9}{13}$

Then
$(ab^2 + a^2b + 1)\ S$

$(a^5, a^2 \times a^2 + 1)\ \frac{9}{13}$

$(243 + 81 + 1)\ \frac{9}{13}$

= 225

Sol.19 $A^{-2} = A^{-1} \times A^{-1}$

$A = \begin{bmatrix} 4 & -4 & 5 \\ -2 & 3 & -3 \\ 3 & -3 & 4 \end{bmatrix}$ $CA = \begin{bmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ -3 & 2 & 4 \end{bmatrix}$

$\text{Adj } A = (cij)^T = \begin{bmatrix} 3 & 1 & 3 \\ -1 & 1 & 2 \\ -3 & 0 & 4 \end{bmatrix}$

$|A| = \begin{bmatrix} 4 & -4 & 5 \\ -2 & 3 & -3 \\ 3 & -3 & 4 \end{bmatrix}$

= 4(3) + 4 – 15
= 16 – 15

= 1 $A + \frac{\text{Adj}(A)}{|A|}$

$A^{-1} = \begin{bmatrix} 3 & 1 & 3 \\ -1 & 1 & 2 \\ -3 & 0 & 4 \end{bmatrix}$

$A^{-1} A^{-1} = \begin{bmatrix} 3 & 1 & 3 \\ -1 & 1 & 2 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 & 3 \\ -1 & 1 & 2 \\ -3 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 17 & 4 & -19 \\ -10 & 0 & 13 \\ -21 & -3 & 25 \end{bmatrix}$

Sol.20 BPA = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} P \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \; 2 \times 3$$

order of P will be 2 × 3

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$$

Sol.21 **(a)** $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

Cofacter elements matrix = $\begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Adj A = $(cij)^T$ = $\begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

(b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$

$$C_{11} = \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & \omega \end{vmatrix} = \omega^2 - \omega^4 = (\omega^2 - \omega)$$

$$C_{12} = (-1) \begin{vmatrix} 1 & \omega^2 \\ 1 & \omega \end{vmatrix} = \omega^2 - \omega$$

$$C_{13} = \begin{vmatrix} 1 & \omega \\ 1 & \omega^2 \end{vmatrix} = \omega^2 - \omega$$

$$C_{21} = (-1) \begin{vmatrix} 1 & 1 \\ \omega^2 & \omega \end{vmatrix} = \omega^2 - \omega$$

$$C_{22} = \begin{vmatrix} 1 & 1 \\ 1 & \omega \end{vmatrix} = \omega - 1$$

$$C_{23} = (-1) \begin{vmatrix} 1 & 1 \\ 1 & \omega^2 \end{vmatrix} = 1 - \omega^2$$

$$C_{31} = \begin{vmatrix} 1 & 1 \\ \omega & \omega^2 \end{vmatrix} = \omega^2 - \omega$$

$$C_{32} = (-1) \begin{vmatrix} 1 & 1 \\ 1 & \omega^2 \end{vmatrix} = (\omega^2 - 1) = 1 - \omega^2$$

$$C_{33} = \begin{vmatrix} 1 & 1 \\ 1 & \omega \end{vmatrix} = \omega - 1$$

$$C = \begin{bmatrix} \omega^2 - \omega & \omega^2 - \omega & \omega^2 - \omega \\ \omega^2 - \omega & \omega - 1 & 1 - \omega^2 \\ \omega^2 - \omega & -(\omega^2 - 1) & \omega - 1 \end{bmatrix}$$

$$\text{adj } A = c^T = \begin{bmatrix} \omega^2 - \omega & \omega^2 - \omega & \omega^2 - \omega \\ \omega^2 - \omega & \omega - 1 & -(\omega^2 - 1) \\ \omega^2 - \omega & 1 - \omega^2 & \omega - 1 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \Rightarrow 1\ \{\omega - \omega\} - 1\ \{\omega - \omega\} + 1\ \{\omega^2 - \omega\}$$

$$\Rightarrow \omega^2 - \omega - \omega + \omega^2 + \omega^2 - \omega$$

$$\Rightarrow 3(\omega^2 - \omega) \quad \Rightarrow \quad 3\,\omega\,(\omega - 1)$$

$$A^{-1} = \frac{\text{adj. } A}{|A|}$$

$$= \frac{1}{3\cos(\omega - 1)} \begin{bmatrix} \omega(\omega-1) & \omega(\omega-1) & \omega(\omega-1) \\ \omega(\omega-1) & \omega - 1 & -(\omega-1)\omega + 1 \\ \omega(\omega-1) & (1-\omega)(1+\omega) & \omega - 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \frac{1}{\omega} & -(\omega + 1) \\ 1 & -(1+\omega) & \frac{1}{\omega} \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & +\omega^2 \\ 1 & +\omega^2 & \omega^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(c) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ for any diagonal

matrix inverse of a diagonal matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \text{ then}$$

$$A^{-1} = \begin{bmatrix} 1/A_{11} & 0 & 0 \\ 0 & 1/A_{22} & 0 \\ 0 & 0 & 1/A_{33} \end{bmatrix}$$

$$\text{so } A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$$

Sol.22 A B

$$\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{\text{Adj}(B)}{|B|}$$

$$\text{Adj (B)} = \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$$

$$|B| = 1 + \tan^2 \theta/2$$
$$= \sec^2 \theta/2$$

$$B^{-1} = \begin{bmatrix} \cos^2\theta/2 & \dfrac{-\tan\theta/2}{\sec^2\theta/2} \\ \dfrac{\tan\theta/2}{\sec^2\theta/2} & \dfrac{1}{\sec^2\theta/2} \end{bmatrix}$$

$$A\,B^{-1} = \begin{bmatrix} 1 & -\text{tn}\,\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\theta/2 & \dfrac{-\tan\theta/2}{\sec^2\theta/2} \\ \dfrac{\tan\theta/2}{\sec^2\theta/2} & \dfrac{1}{\sec^1\theta} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\tan\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Sol.21 **(i)** $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$\text{Cofacter elements matrix} = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\text{Adj A} = (\text{cij})^T = \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$A^{-1} = \frac{\text{Adj A}}{|A|} = \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

(b) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ for any king materix

inverce of agovel matrix

$$iJ = A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \text{ then}$$

$$A^{-1} = \begin{bmatrix} 1/A_{11} & 0 & 0 \\ 0 & 1/A_{22} & 0 \\ 0 & 0 & 1/A_{33} \end{bmatrix}$$

$$\text{so } A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$$

Sol.23 $F(x + y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\sin x\cos y - \cos x\sin y & 0 \\ \sin x\cos y + \cos x\sin y & \cos x\cos y - \sin x\sin y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

= F(x) F(y)

Now $F(x) . F(-x) = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= I$

$F(x) F(-x) = I$

$\Rightarrow [F(x)]^{-1} = F(-x)$ Hence proved

Sol.24 A is skew symmetric matrix

$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$

$(I + A) = \begin{bmatrix} 1 & 5 \\ -5 & 1 \end{bmatrix}$ $|I + A| = 1 + 25 = 26$

$|I + A|^{-1} = \dfrac{Adj(I + A)}{|I + A|}$

$= 1/26 \begin{bmatrix} 1 & 5 \\ -5 & 1 \end{bmatrix}$

$[I + A]^{-1} [I - A] = 1/26 \begin{bmatrix} 1 & 5 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & -5 \\ 5 & 1 \end{bmatrix}$

$= 1/26 \begin{bmatrix} 26 & 0 \\ 0 & 26 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is

Sol.25 **(i)** $x + y + z = 3$
$x + 2y + 3y = 4$
$x + 4y + 9z = 6$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix} = -6$

$D \neq 0$

$x = \dfrac{D_1}{D}, \; y = \dfrac{D_2}{D}, \; z \mp \dfrac{D_3}{D}$

$D_1 = \begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 3 \\ 6 & 4 & 9 \end{vmatrix}$ $D_2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 4 & 3 \\ 1 & 6 & 9 \end{vmatrix}$ $D_3 = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 4 & 6 \end{vmatrix}$

$x = 2$
$y = 1$
$z = 0$

(ii) $x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$
$D \neq 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 1(-2) - 1 + 1(3)$

$= -2 + 1 + 3$
$= 2$

$x = \dfrac{D_1}{D}, \; y = \dfrac{D_2}{D}, \; z = \dfrac{D_3}{D}$

$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$

$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$

(b) $x + y + z = 3$
$x + 2y + 3z = 4$
$2x + 3y + 4z = 9$
$D \neq 0$
$D = 0$
No solution
$D_1 = D_2 = D_3 = 0$

$D_1 = \begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 3 \\ 9 & 3 & 4 \end{vmatrix} = 3(8-9) - 1(16-27) + 1(12-1)$

= no zero

$D_2 = |$
$D_1 \neq 0$ so no solution

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{bmatrix}, \; C = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \; D = \begin{bmatrix} 10 \\ 13 \\ 9 \end{bmatrix}$

Cb = D Let $b = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \\ 9 \end{bmatrix}$

$x = 1$
$y = 3$
$z = 5$
$Ax = b$

$$\begin{bmatrix}1 & 2 & 2\\2 & 2 & 3\\1 & -1 & 3\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}1\\3\\5\end{bmatrix}$$

$$\begin{bmatrix}a=1\\b=-1\\c-1\end{bmatrix} \qquad x = \begin{bmatrix}1\\-1\\1\end{bmatrix}$$

Sol.26 $\begin{bmatrix}2 & 1 & 1\\2 & 2 & 1\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}10\\13\\9\end{bmatrix}$

$x = \frac{D_1}{D}$; $y = \frac{D_2}{D}$; $z = \frac{D_3}{D}$

$\Rightarrow$ x = 1, y = 3, z = 5

$$\begin{bmatrix}1 & 2 & 2\\2 & 2 & 3\\1 & -1 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\3\\5\end{bmatrix}$$

$\Rightarrow$ multiply & compare

$x_1 = 1$; $x_2 = -1$; $x_3 = 1$

Sol.27 $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix} A\begin{bmatrix}3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}2 & 4\\3 & -1\end{bmatrix}$

$$A = \begin{bmatrix}x & y\\z & w\end{bmatrix}$$

$$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}\begin{bmatrix}x & y\\z & w\end{bmatrix}\begin{bmatrix}3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}-2 & 4\\3 & -1\end{bmatrix}$$

$x = \frac{48}{19} \qquad y = \frac{-25}{19}$

$z = \frac{-70}{19} \qquad w = \frac{42}{19}$

Sol.28 $A = \begin{bmatrix}k & m\\\lambda & n\end{bmatrix}$ $kn \neq \lambda m$

$A^2 - (k + 4) A + (kn - \lambda m) I =$

$$A^2 = \begin{bmatrix}k & m\\\lambda & n\end{bmatrix}\begin{bmatrix}k & m\\\lambda & n\end{bmatrix} = \begin{bmatrix}k^2 + m\ell & km + mn\\k\ell + n\ell & m\ell + n^2\end{bmatrix}$$

$$\begin{bmatrix}k^2 + m\ell & km + mn\\k\ell + n\ell & m\ell + n^2\end{bmatrix} - (k + n)\begin{bmatrix}k & m\\\ell & n\end{bmatrix}$$

$$+ \begin{bmatrix}kn - \ell n & 0\\ & kn - \ell n\end{bmatrix} = 0$$

$A^2 - (k + n) A + (kn - \lambda n) I = 0 \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ proceed

Multiply by A^{-1}

$A - (k + n) I + (kn - lm) A^{-1} = A^{-1} 0$

$$A^{-1} = \frac{1}{kn - \ell m}\begin{bmatrix}n & -m\\-\ell & k\end{bmatrix}$$

Sol.29 **(a)** $A = \begin{bmatrix}2 & 1\\2 & 1\end{bmatrix}$ $B = \begin{bmatrix}9 & 3\\3 & 1\end{bmatrix}$

(i) AX = A

$$\begin{bmatrix}2 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}2 & 1\\2 & 1\end{bmatrix}$$

$$\begin{bmatrix}2a + c & 2b + d\\2a + c & 2b + d\end{bmatrix} = \begin{bmatrix}2 & 1\\2 & 1\end{bmatrix}$$

2a + c .....(1)

2b + d = 1 .....(2)

c = 2 – 2a

d = 1 – 2b

$$\begin{bmatrix}a & b\\2 - 2a & 1 - 2b\end{bmatrix}$$

(b) A = I

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}2 & 1\\2 & 1\end{bmatrix} = I$$

$$\begin{bmatrix}2a + 2d & a + d\\2c + 2d & c + d\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\left.\begin{array}{l}2(a + b) = 1\\a + b = 0\\2(c + d) = 1\\c + d = 0\end{array}\right| \text{Does not exist}$$

(c) XB = 0 but $Bx \neq 0$

$$\begin{bmatrix}a & b\\c & a\end{bmatrix}\begin{bmatrix}a & 3\\3 & 1\end{bmatrix}$$

$$\begin{bmatrix}9a + 3b & 3a + b\\9c + 3d & 3c + d\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$Bx \neq 0$

9a + 3b = 0

3a + b = 0

ac + 3d = 0

3c + d = 0

$$\begin{bmatrix}9 & 3\\3 & 1\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\begin{bmatrix}9a + 3c & 9b + 3d\\3a + c & 3b + d\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$\begin{bmatrix}a & -3a\\c & -3c\end{bmatrix}$ $3a + c \neq 0$

Sol.30 $AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I$

$x + y + 2z = 1$
$3x + 2y + z = 7$
$2x + 2y + 3z = 2$

$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}_{3\times 1}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

Sol.31 $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 0$

$\begin{bmatrix} a+2c & b+2d \\ 2a+4c & 2b+4d \end{bmatrix} = 0$

$a + 2c = 0 \qquad b + 2d = 0$
$a = -2c \qquad b = -2d$

$x = \begin{bmatrix} -2c & -2d \\ c & d \end{bmatrix}$

Sol.32 $\begin{bmatrix} 3 & -2 & 1 \\ 5 & 8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$

(i) unique solution

$\begin{vmatrix} 3 & -2 & 1 \\ 5 & 8 & 9 \\ 2 & 1 & a \end{vmatrix} \neq 0$

$a \neq 3$

(ii) has no solution

D = 0 so a should a = 3 at least one form D_1, D_2, D_3 non zero $b \neq 1/3$

$\begin{vmatrix} b & -2 & 1 \\ 3 & -8 & 9 \\ -1 & 1 & 3 \end{vmatrix} \neq 0$

Sol.33 **(a)** $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$x = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$

(b) $(B - I) \times = IC$

$\left(\begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\begin{bmatrix} 2x_1 + x_2 & 2x_3 + x_4 \\ x_1 - x_2 & x_3 - x_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$x = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

(c) $Cx = A$

$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\begin{bmatrix} x_1 + 2x_3 & 2x_1 + 4x_3 \\ x_2 + 2x_4 & 2x_2 + 4x_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$x_1 + 2x_3 = 1$
$2x_1 + 4x_2 = 2$ No solution

Sol.34 $B = AP \qquad A^T A = I$
$BB^{-1} = AP\, B^{-1}$
$I = APB^{-1}$
$(AA^T)^T = (APB^{-1})^T$
$AA^T = (B^{-1})^T (P)^T A^T$
$A = (B^{-1})^T (P)^T$
$A^T = PB^{-1}$
$(PB^{-1}) (PB^{-1})^T = I$
So PB^{-1} is orthogonal